INTRODUCTION OF LEARNING SEMINAR ON QUANTUM TOPOLOGY 2022

During this season of the learning seminar, we will focus on the study of **quantum invariants of 3-manifolds** by using **Temperley-Lieb algebras**. We will follow closely the book:

Temperley-Lieb Recoupling Theory and Invariants of 3-Manifolds
(Louis H. Kauffman and Sóstenes L. Lins)

Our main goal is to understand the definition of **Turaev-Viro** and **Reshetikhin-Turaev** invariants and their explicit computation. In particular, we will learn:

- Kauffman bracket and Temperley-Lieb algebras.
- Triangulations, spines and Heegaard splittings of 3-manifolds.
- Turaev-Viro invariants.
- Reshetikhin-Turaev invariants.

Below there is a more detailed introduction.

1. Detailed introduction

For topological 3-manifolds, there are basically two kinds of invariants; the so-called *classical invariants* (e.g. homotopy groups, (co)homology groups) and *quantum invariants* (e.g. Turaev-Viro invariants, Witten-Reshetikhin-Turaev invariants). In general, to construct (quantum) invariants of 3-manifolds, the scheme is as follows:

$$\begin{array}{c}\text{Combinatorial presentation}\\ \text{of 3-manifolds}\\ +\\ \text{Some data}\end{array} \quad \Longrightarrow \quad \text{Invariants}$$

Given the combinatorial presentation, the homeomorphism condition is translated into **combinatorial moves**, thus in order to get an invariant I of 3-manifolds, it is enough to show the invariance of I under these moves.

The study of quantum invariants of knots and 3-manifolds has its origins in the 1980s with V. Jones' discovery of his polynomial knot invariant and E. Witten's physical interpretation of the Jones polynomial. This interpretation allowed Witten to propose, at the physics level of rigor, new invariants of 3-manifolds. A mathematical definition of Witten's invariants was first obtained by

- N. Reshetikhin and V. Turaev, using surgery presentations of 3-manifolds and representations of quantum groups.
- V. Turaev and O. Viro, using triangulations (and spines) of 3-manifolds and representations of quantum groups.

The original definition of the Jones polynomial uses as the first step a representation of braids groups on **Temperley-Lieb algebras**. The **Kauffman bracket** reformulation of the Jones polynomial allows to give a purely diagrammatic interpretation of Temperley-Lieb algebras which we will study in detail. We will also see that the theory of Temperley-Lieb algebras gives us the necessary data for the definition of Turaev-Viro and Reshetikhin-Turaev invariants.